Beta-neutrino angular-correlation coefficient in ^{21}Na

I. S. Towner and J. C. Hardy

Recently [1], an 'exact' calculation of the beta-neutrino angular-correlation coefficient for ^{21}Na was published. The coefficient is defined as

$$a_{ev}(W) = a_{ev}^0 + \Delta a_{ev}(W) \tag{1}$$

with $a_{ev}^0 = (a_1^2 - \frac{1}{3}c_1^2)/(a_1^2 + c_1^2)$ being the major contribution, where $a_1 = g_V M_F$ and $c_1 = g_A M_{GT}$ with M_F and M_{GT} being the Fermi and Gamow-Teller matrix elements and g_V and g_A their respective coupling constants. Here W is the electron total energy expressed in electron rest-mass units. We computed the correction Δa_{ev} using the exact formalism of Behrens and Bühring (BB) [2] and found Δa_{ev}, when averaged over the electron energy spectrum, gave a correction of order 1%. Alternatively computing the correction with the formalism of Holstein [3] we found the correction to be much smaller, of order 0.05%. To try and resolve this discrepancy we have been working with the BB formalism, identifying the leading order terms and comparing them with those of Holstein.

The beta-decay differential decay rate is written as

$$d^5\Gamma = \frac{G^2}{(2\pi)^5} F(Z,W)(W_0 - W)^2 pW dW d\Omega_e d\Omega_\nu \left(f_1(W) + f_2(W)\frac{p}{w}\hat{\boldsymbol{p}}.\hat{\boldsymbol{k}} + \cdots\right) \tag{2}$$

where $\hat{\boldsymbol{p}}$ and $\hat{\boldsymbol{k}}$ are unit vectors in the directions of the electron and neutrino respectively. Here W_0 is the maximum value of W, p the electron momentum $p^2 = W^2 - 1$ in electron rest-mass units and $F(Z,W)$ the Fermi function. The beta-neutrino angular-correlation coefficient is defined as

$$a_{ev} = \frac{f_2(W)}{f_1(W)} \tag{3}$$

Holstein [3] gives expressions for the spectral functions with electromagnetic corrections to order (αZ). It is convenient to write out separately the Fermi and Gamow-Teller pieces, $f_1(W) = f_1^F(W) + f_1^{GT}(W)$ and $f_2(W) = f_2^{GT}(W) + f_2^{GT}(W)$, and show their electron energy dependence explicitly by writing them as

$$f_\alpha^F(W) = a_1^2 k_\alpha^F \left(1 + A_\alpha^F W + \frac{B_\alpha^F}{W} + C_\alpha^F W^2\right)$$
$$f_\alpha^{GT}(W) = c_1^2 k_\alpha^{GT} \left(1 + A_\alpha^{GT} W + \frac{B_\alpha^{GT}}{W} + C_\alpha^{GT} W^2\right), \qquad \alpha = 1,2 \tag{4}$$

For a mixed Fermi plus Gamow-Teller transition, these expressions are combined by defining

$$f_\alpha(W) = (a_1^2 + c_1^2)k_\alpha \left(1 + A_\alpha W + \frac{B_\alpha}{W} + C_\alpha W^2\right), \quad \alpha = 1, 2 \tag{5}$$

where, for example, $A_\alpha = (a_1^2 A_\alpha^F + c_1^2 A_\alpha^{GT})/(a_1^2 + c_1^2)$ and likewise for k_α, B_α and C_α.

In comparing our results from BB with the expressions of Holstein, we note two main differences:

- There is no electromagnetic (αZ) correction to Holstein's weak-magnetism form factor;
- Behrens-Bühring argue that (W_0R) and (αZ) are comparable small quantities and expressions evaluated to second order should contain terms in $(W_0R)^2$, $(W_0R)(\alpha Z)$ and $(\alpha Z)^2$. Holstein does not provide any corrections in order $(\alpha Z)^2$.

These differences do not represent errors, but reflect different approximations made in the derivations. There is little numerical consequence from the second item in this list. It is the electromagnetic correction to the weak-magnetism form factor that explains the differing beta-neutrino angular correlation values in ^{21}Na.

To illustrate this, we perform a shell-model calculation with the USD effective interaction [4] of the nuclear matrix elements involved in ^{21}Na beta decay. Note that the Gamow-Teller form factor, c_1, can alternatively be determined from the experimental *ft* value using $ft = \frac{6140}{B(GT)}$ and $|c_1| = \sqrt{B(GT)}$. Thus we have adjusted the g_A value so that when combined with the shell-model matrix element M_{GT} the experimental c_1 value is obtained. With these shell-model values we compute the beta-neutrino correlation coefficient, a_{ev}, (averaged over the electron energy spectrum) with the formulae from Eq. (4) and 'exactly' using a computer code [5] based on the Behrens-Büring formalism. In the exact computation, the spectral functions $f_1(W)$ and $f_2(W)$ are obtained in numerical form. We therefore use a least-squares fitting program to fit these tabular values to the expressions given in Eq. (4). In the fitting only two parameters could be usefully determined; so for $f_1(W)$ we fixed B_1 and C_1 to the values given by the formulae and determined k_1 and A_1. Likewise for $f_2(W)$, we fixed B_2 and C_2 and determined k_2 and A_2. The results are given in Table I. One sees the Behrens-Bühring formulae agree well with the 'exact'

Table I. Values of the coefficients in the spectral functions $\boldsymbol{f_1(W)}$ and $\boldsymbol{f_2(W)}$, Eq.(5), for the beta decay of ^{21}Na obtained from the formulae of Holstein and Behrens-Bühring and the 'exact' computation.

	k_1	k_1A_1(%)	k_1B_1(%)	k_1C_1(%)		
Holstein	1.00417	-0.027	0.115	-0.003		
BB formula	1.01133	-0.027	0.115	-0.003		
BB'exact'	1.01066	-0.020				
	k_2	k_2A_2(%)	k_2B_2(%)	k_2C_2(%)	a_{ev}	Δa_{ev}
Holstein	0.55489	0.188	0.000	0.002	0.55867	-0.00028
BB formula	0.55296	0.187	0.020	0.002	0.55286	-0.00592
BB'exact'	0.55243	0.195			0.55330	-0.00548

computed values, while the Holstein formulae appear deficient in the energy-independent coefficients k_1 and k_2. This we trace to the absence of the electromagnetic weak-magnetism term in the Holstein formulae.

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